Trigonometry for Calculus

Presented by the Quantitative Success Center

RECIPROCAL IDENTITIES

$$\sin(\theta) = \frac{1}{\csc(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Quotient Identities

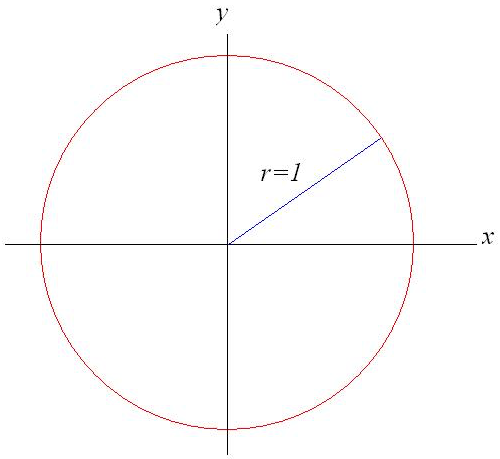
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Remember: ALL identities can be written in terms of _____ & _____.

A. Unit circle

Recall: $x = \cos\theta\,, y = \sin\theta$
where θ is the angle you take going counterclockwise from the positive x-axis.

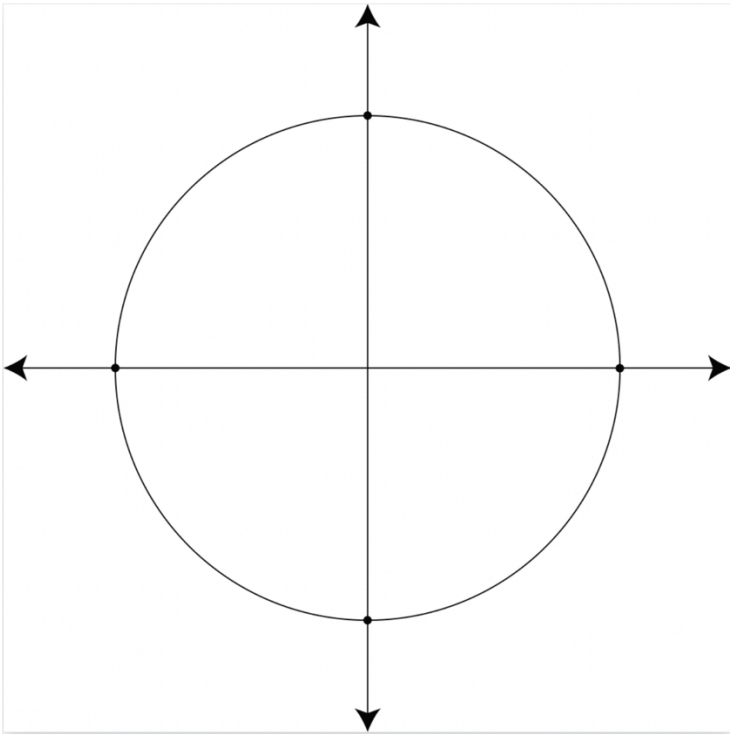


Quadrant	Are the x-values positive or negative?	Are the y-values positive or negative?	Are the $\frac{y}{x}$ values positive or negative?
I			
II			
III			
IV			

Find the sign of each trigonometric function in the respective quadrant.

Quadrant	$\cos x$	$\sin x$	$\tan x$	$\sec x$	$\csc x$	$\cot x$
I						
II						
III						
IV						

We can conclude - **A**ll **S**tudents **T**ake **C**alculus:



B. Pythagorean Identities

(Manipulating $\cos^2\theta + \sin^2\theta = 1$ to get the other identities)

a. Since $x^2 + y^2 = 1$ on the unit circle, we get $\cos^2\theta + \sin^2\theta = 1$

b. Let's divide our identity from part a) by $\cos^2\theta$ and see what we get:

c. Let's divide our identity from part a) by $\sin^2\theta$ and see what we get:

d. We can conclude with the three trigonometric identities from (a)-(c):

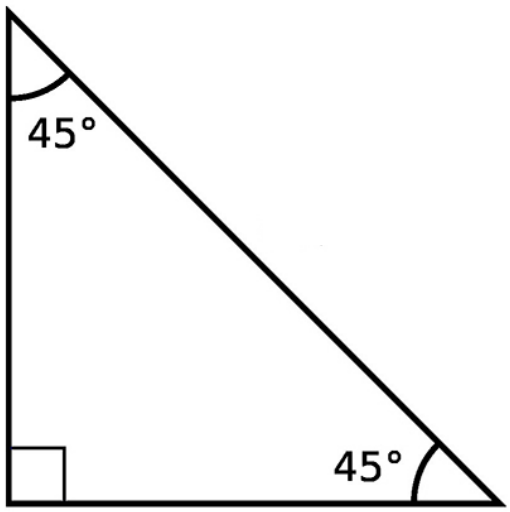
C. Converting between degrees and radians.

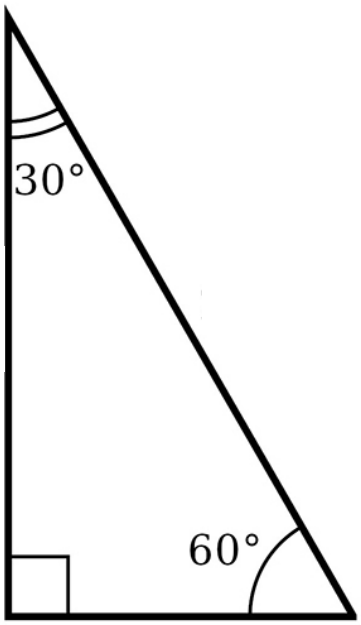
Recall $\pi = 180°$

Degrees°	Radians
$30°$	
	$\frac{\pi}{4}$
$60°$	
	$\frac{\pi}{2}$

Degrees°	Radians
$120°$	
$135°$	
	$\frac{4\pi}{3}$
	$\frac{11\pi}{6}$

D. Special Right Triangles & Reference Angles





Recall: SOH CAH TOA. $\sin x = \frac{opposite}{hypotenuse}$, $\cos x = \frac{adjacent}{hypotenuse}$, $\tan x = \frac{opposite}{adjacent}$

a. What is the reference angle for $\frac{2\pi}{3}$?

b. What is the reference angle for $\frac{3\pi}{4}$?

c. What is the reference angle for $240°$?

d. What is the reference angle for $330°$?

More Practice:

Find the exact values using unit circle/triangles/identities.

1. 30°

$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

2. $\frac{\pi}{4}$

$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

3. 60°

$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

4. $\frac{\pi}{2}$

$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

5. 120°

$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

6. $\frac{4\pi}{3}$

$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

7. 135°

$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

8. $\frac{11\pi}{6}$

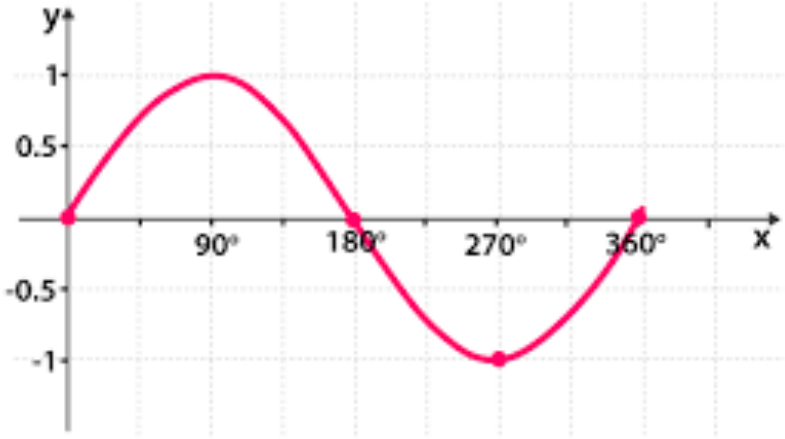
$\cos x =$	$\sec x =$
$\sin x =$	$\csc x =$
$\tan x =$	$\cot x =$

E. Determine the exact values for the following trigonometric functions when the exact angles are not given:

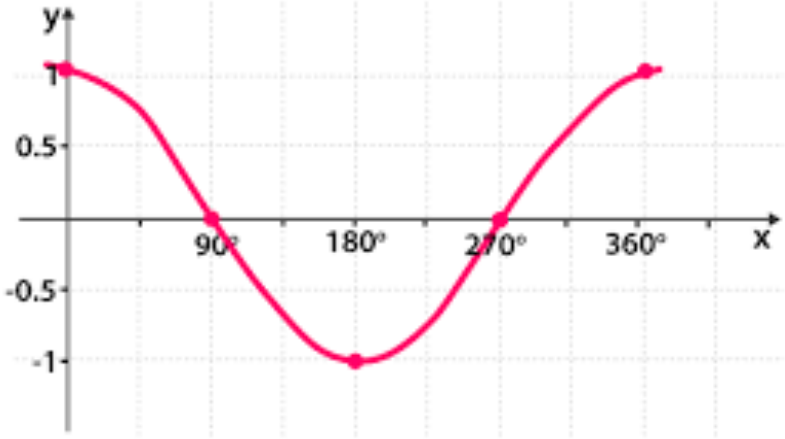
$\sin x$	$\cos x$	$\tan x$
$\frac{2}{3}$		
		$\frac{1}{4}$
	$\frac{4}{5}$	

F. Graphs of sine and cosine to also help determine exact values

a. Graph of $y = \sin x$



b. Graph of $y = \cos x$



G. More resources

- For the review and/or its solutions, visit qsc.whittier.domains and click on "Workshops"
- https://tutorial.math.lamar.edu/classes/calci/calci.aspx

Workshop Survey

https://forms.gle/y6u2s8TQymPYA1vN9

More QSC Workshops