Key

Trigonometry for Calculus

Presented by the Quantitative Success Center

RECIPROCAL IDENTITIES

$$\sin(\theta) = \frac{1}{\csc(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Quotient Identities

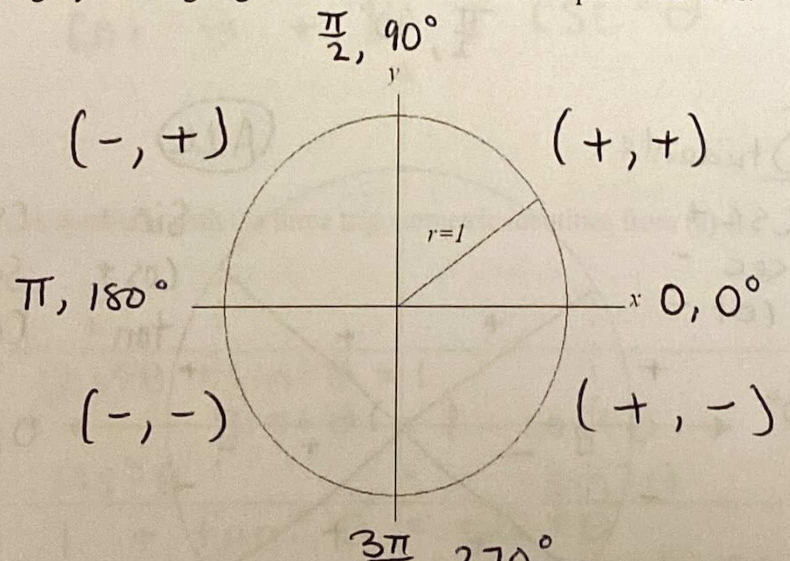
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Remember: ALL identities can be written in terms of sinθ **&** cosθ **.**

A. Unit circle

Recall: $x = \cos\theta$, $y = \sin\theta$
where θ is the angle you take going counterclockwise from the positive x-axis.



Quadrant	Are the x-values positive or negative?	Are the y-values positive or negative?	Are the $\frac{y}{x}$ values positive or negative?
I	+	+	+
II	−	+	−
III	−	−	+
IV	+	−	−

Key

Find the sign of each trigonometric function in the respective quadrant.

Quadrant	$\cos x$	$\sin x$	$\tan x$	$\sec x$	$\csc x$	$\cot x$
I	+	+	+	+	+	+
II	−	+	−	−	+	−
III	−	−	+	−	−	+
IV	+	−	−	+	−	−

*hypotenuse is always positive

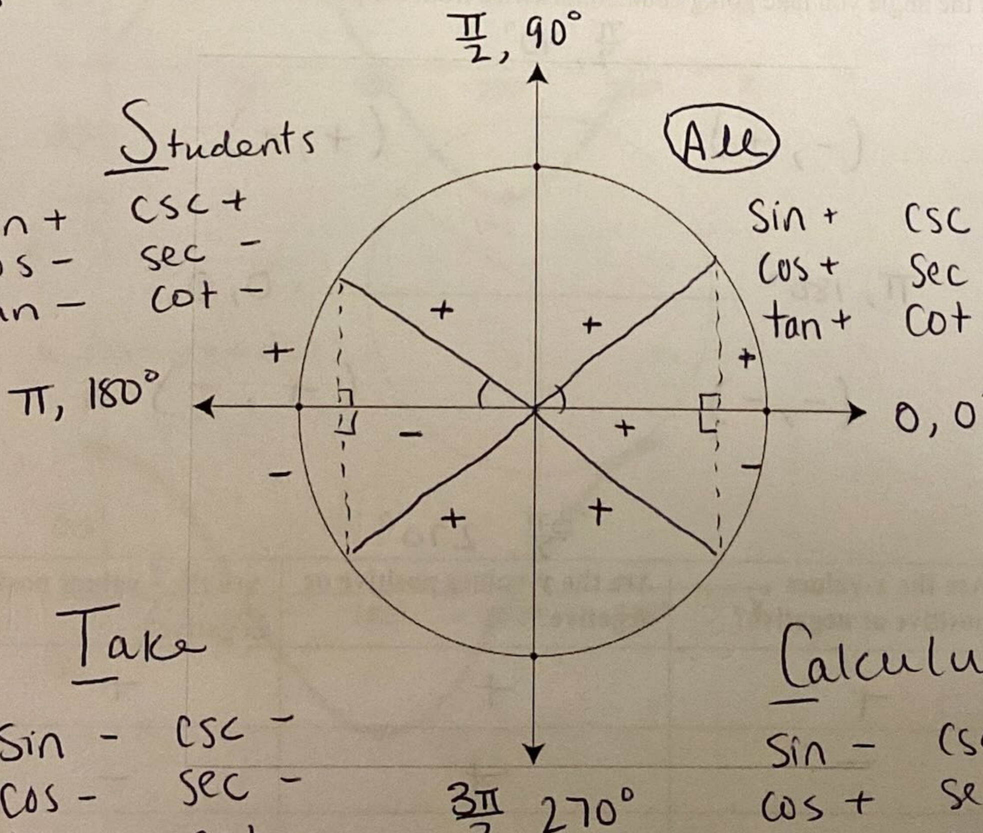
We can conclude - **A**ll **S**tudents **T**ake **C**alculus:

Students

Sin + csc +
cos − sec −
tan − cot −

All

Sin + csc +
cos + Sec +
tan + Cot +



Take

Sin − csc −
cos − sec −
tan + cot +

Calculus

Sin − csc −
cos + sec +
tan − Cot −

B. Pythagorean Identities

(Manipulating $\cos^2\theta + \sin^2\theta = 1$ to get the other identities)

can rearrange & solve for any part

i.e. $\sin^2\theta = 1 - \cos^2\theta$

a. Since $x^2 + y^2 = 1$ on the unit circle, we get $\cos^2\theta + \sin^2\theta = 1$

b. Let's divide our identity from part a) by $\cos^2\theta$ and see what we get:

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

c. Let's divide our identity from part a) by $\sin^2\theta$ and see what we get:

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\cot^2\theta + 1 = \csc^2\theta$$

d. We can conclude with the three trigonometric identities from (a)-(c):

a)	$\cos^2\theta + \sin^2\theta = 1$ $\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$
b)	$1 + \tan^2\theta = \sec^2\theta$ $1 = \sec^2\theta - \tan^2\theta$ $\tan^2\theta = \sec^2\theta - 1$
c)	$\cot^2\theta + 1 = \csc^2\theta$ $1 = \csc^2\theta - \cot^2\theta$ $\cot^2\theta = \csc^2\theta - 1$

C. Converting between degrees and radians.
Recall $\pi = 180°$

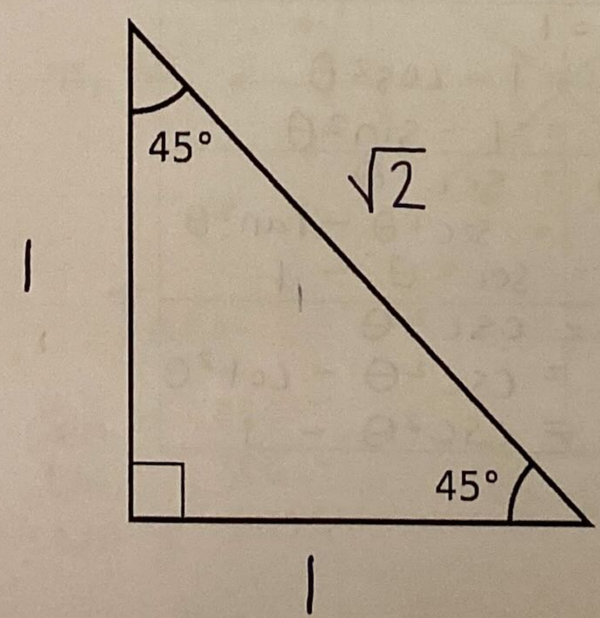
ex: $\frac{30°}{1} \times \frac{\pi}{180°} = \frac{30\pi}{180} = \frac{\pi}{6}$

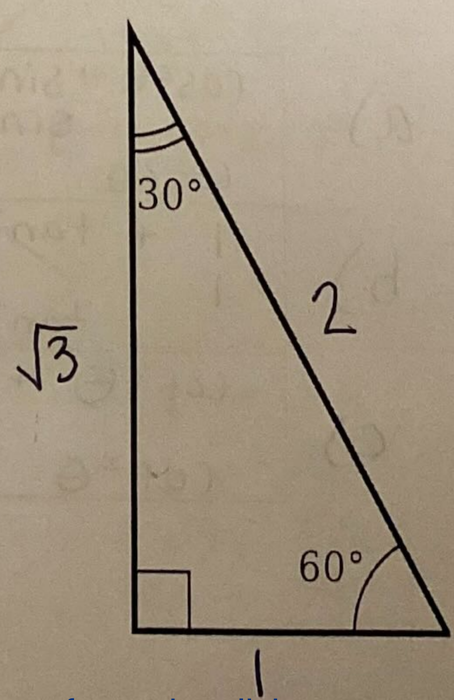
ex: $\frac{4\pi}{3} \times \frac{180°}{\pi} = \frac{4 \times 180°}{3} = 240°$

Degrees°	Radians
30°	$\frac{\pi}{6}$
45°	$\frac{\pi}{4} \cdot \frac{180}{\pi} = \frac{180°}{4}$
$\frac{60°}{1} \cdot \frac{\pi}{180°}$	$\frac{\pi}{3}$
90°	$\frac{\pi}{2} \cdot \frac{180°}{\pi} = \frac{180°}{2}$

Degrees°	Radians
$\frac{120°}{1} \cdot \frac{\pi}{180°} =$	$\frac{2\pi}{3}$
$\frac{135°}{1} \cdot \frac{\pi}{180°}$	$\frac{3\pi}{4}$
240°	$\frac{4\pi}{3}$
330°	$\frac{11\pi}{6} \cdot \frac{180}{\pi} = \frac{11 \cdot 180°}{6}$

D. Special Right Triangles & Reference Angles

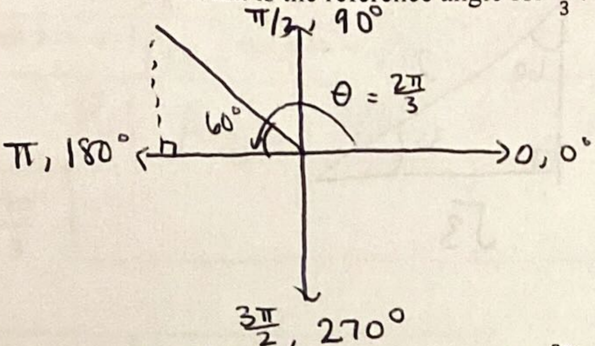




Note: We still get the same answers after rationalizing
if we use 1/2 as the length across 30 degrees,
(sqrt2)/2 as the length across 45 degrees, (sqrt3)/2 as the length across 60 degrees,
and (sqrt4)/2=2/2=1 as the length across 90 degrees

Recall: SOH CAH TOA. $\sin x = \frac{opposite}{hypotenuse}$, $\cos x = \frac{adjacent}{hypotenuse}$, $\tan x = \frac{opposite}{adjacent}$

a. What is the reference angle for $\frac{2\pi}{3}$?



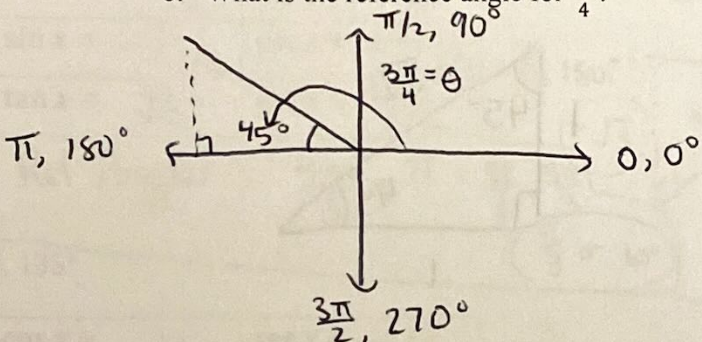
Ref. angle $= \pi - \frac{2\pi}{3}$

$\frac{3\pi}{3} - \frac{2\pi}{3} = \boxed{\frac{\pi}{3}}$

OR

$\boxed{60^\circ}$

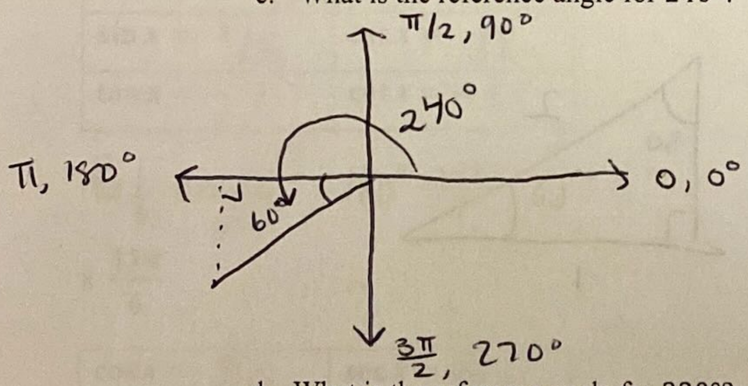
b. What is the reference angle for $\frac{3\pi}{4}$?



Ref. Angle $= \pi - \frac{3\pi}{4}$

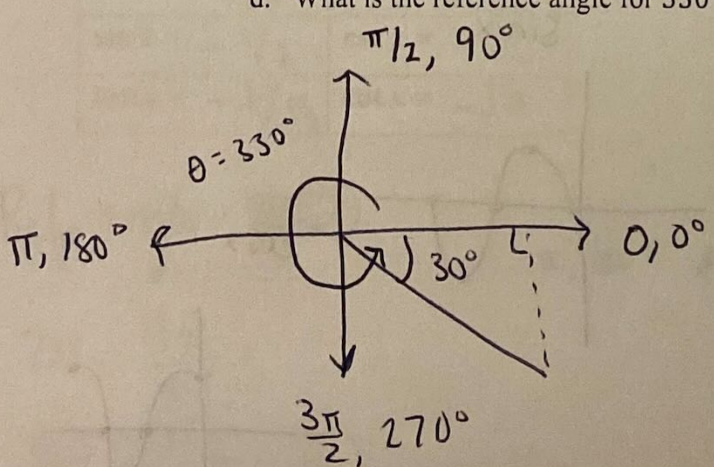
$\frac{4\pi}{4} - \frac{3\pi}{4} = \boxed{\frac{\pi}{4} \text{ or } 45^\circ}$

c. What is the reference angle for 240°?



Ref. Angle $= 240^\circ - 180^\circ = \boxed{60^\circ \text{ OR } \pi/3}$

d. What is the reference angle for 330°?



Ref Angle =

$360^\circ - 330^\circ = 30^\circ$ OR $\frac{\pi}{6}$

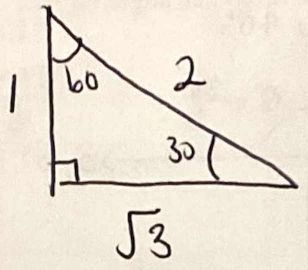
More Practice:

Find the exact values using unit circle/triangles/identities.

OR $\pi/6$

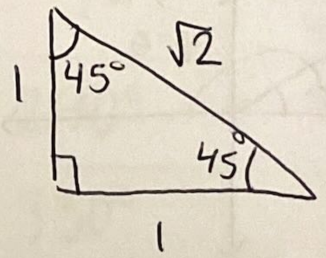
1. 30° all +

$\cos x = \sqrt{3}/2$	$\sec x = 2/\sqrt{3}$
$\sin x = 1/2$	$\csc x = 2$
$\tan x = 1/\sqrt{3}$	$\cot x = \sqrt{3}$



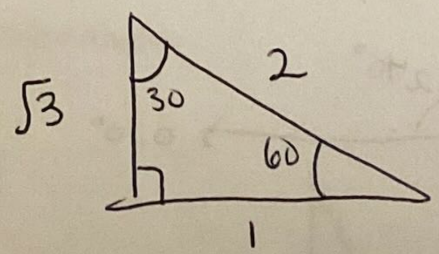
2. $\frac{\pi}{4}$ OR 45°

$\cos x = 1/\sqrt{2}$	$\sec x = \sqrt{2}$
$\sin x = 1/\sqrt{2}$	$\csc x = \sqrt{2}$
$\tan x = 1$	$\cot x = 1$



3. 60° or $\pi/3$

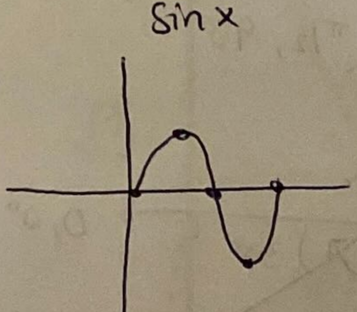
$\cos x = 1/2$	$\sec x = 2$
$\sin x = \sqrt{3}/2$	$\csc x = 2/\sqrt{3}$
$\tan x = 1/\sqrt{3}$	$\cot x = \sqrt{3}$

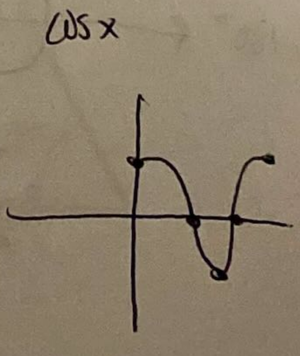


4. $\frac{\pi}{2}$ OR 90

$\cos x = 0$	$\sec x =$ UNDEFINED
$\sin x = 1$	$\csc x = 1$
$\tan x =$	$\cot x = 0$

undefined

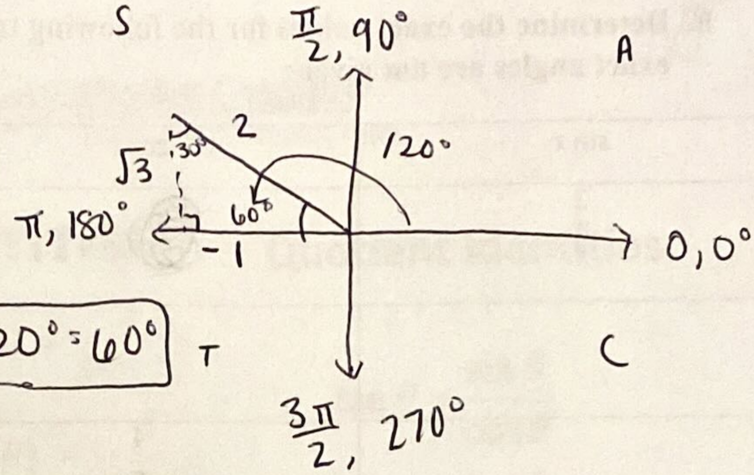




5. 120°

$\cos x = -1/2$	$\sec x = -2$
$\sin x = \sqrt{3}/2$	$\csc x = 2/\sqrt{3}$
$\tan x = -1/\sqrt{3}$	$\cot x = -\sqrt{3}$

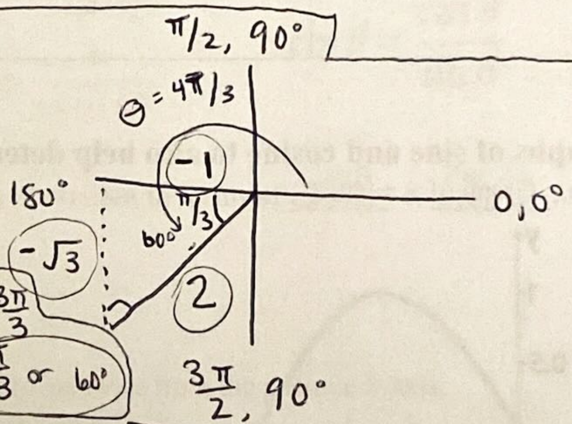
$\boxed{\text{Ref Angle} = 180° - 120° = 60°}$



6. $\frac{4\pi}{3}$

$\cos x = -1/2$	$\sec x = -2$
$\sin x = -\sqrt{3}/2$	$\csc x = -2/\sqrt{3}$
$\tan x = \sqrt{3}$	$\cot x = 1/\sqrt{3}$

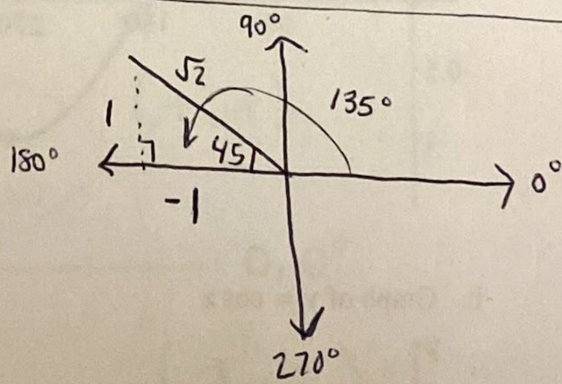
Ref Angle $= \frac{4\pi}{3} - \pi = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$ or 60°



7. 135°

$\cos x = -1/\sqrt{2}$	$\sec x = -\sqrt{2}$
$\sin x = 1/\sqrt{2}$	$\csc x = \sqrt{2}$
$\tan x = -1$	$\cot x = -1$

Ref angle $= 180° - 135° = 45°$



8. $\frac{11\pi}{6}$

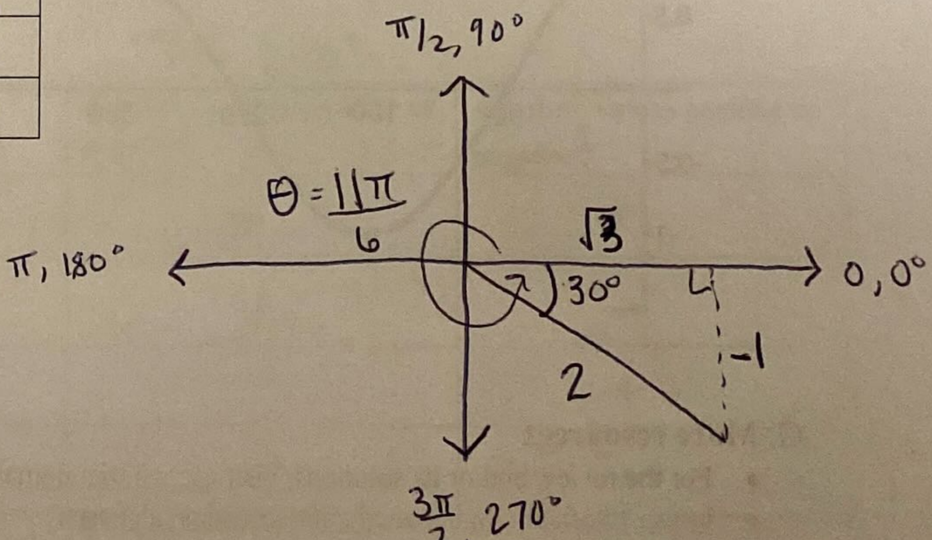
$\cos x = \sqrt{3}/2$	$\sec x = 2/\sqrt{3}$
$\sin x = -1/2$	$\csc x = -2$
$\tan x = -1/\sqrt{3}$	$\cot x = -\sqrt{3}$

Ref angle =

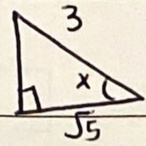
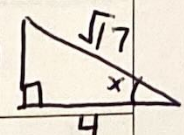
$2\pi - \frac{11\pi}{6} =$

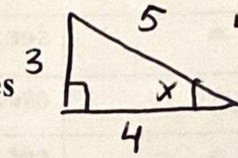
$\frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$

or 30°



E. Determine the exact values for the following trigonometric functions when the exact angles are not given:

$\sin x$	$\cos x$	$\tan x$
$\frac{2}{3}$ $2^2 + b^2 = 3^2$; $b = \sqrt{5}$	 $\cos x = \sqrt{5}/3$	$\tan x = 2/\sqrt{5}$
$1/\sqrt{17}$	$4/\sqrt{17}$	$\frac{1}{4}$ $1^2 + 4^2 = c^2$; $c = \sqrt{17}$ 
$\sin x = 3/5$	$\frac{4}{5}$	$\tan x = 4/5$

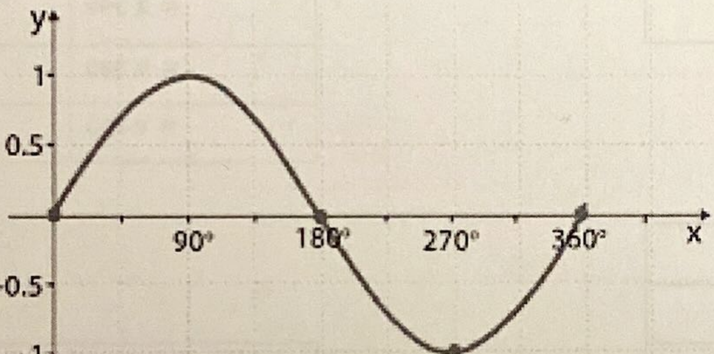


$4^2 + b^2 = 5^2$

$b = 3$

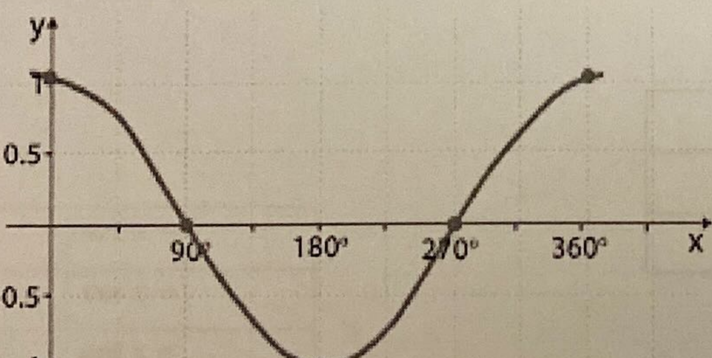
F. Graphs of sine and cosine to also help determine exact values

a. Graph of $y = \sin x$



$\sin(0°) = 0$
$\sin(90°) = 1$
$\sin(180°) = 0$
$\sin(270°) = -1$

b. Graph of $y = \cos x$



$\cos(0) = 1$
$\cos(\pi/2) = 0$
$\cos(\pi) = -1$
$\cos(3\pi/2) = 0$

G. More resources

- For the review and/or its solutions, visit qsc.whittier.domains and click on "Workshops"
- https://tutorial.math.lamar.edu/classes/calci/calci.aspx