Factoring Review

Presented by the Quantitative Success Center

Strategy for factoring polynomials:

Step 1. ***GCF:*** If the polynomial has a greatest common factor other than 1, then factor out the greatest common factor.

Step 2. ***Binomials:*** If the polynomial has two terms (it is a binomial), then see if it is the ***difference of two squares***: $(a^2 - b^2)$.

Remember if it is the sum of two squares, it will NOT factor.

Step 3. ***Trinomials:*** If the polynomial is a trinomial, then check to see if it is a perfect square trinomial which will factor into the square of a binomial: $(a+b)^2$ or $(a-b)^2$.

- If it is not a perfect square trinomial, use factoring **by trial and error** or the AC method.

Strategy for factoring $ax^2 + bx + c$ by grouping (AC method):

a. Form the product ac
b. Find a pair of numbers whose product is ac and whose sum is b.
c. Rewrite the polynomial so that the middle term (bx) is written as the sum of two terms whose coefficients are the two numbers found in step 2.
d. Factor by Grouping (as in step 4)

Step 4. ***Other polynomials:*** If it has more than three terms, try to factor it by grouping.

a. Group two terms together which can be factored further
b. Use the distributive property in reverse to factor out common terms
c. Write the factors as multiplication of binomials.

Step 5. ***Final check:*** See if any of the factors you have written can be factored further. If you have overlooked a common factor. you can catch it here.

Remember the following properties:	
Perfect Squares:	$(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$
Difference of two squares:	$a^2 - b^2 = (a-b)(a+b)$
Sum of two squares:	$a^2 + b^2$ **is NOT factorable**

Sum/Difference of 2 Cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Source:
Chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://www.celinaschools.org/Downloads/Factoring%20Review%20worksheet.pdf

Factoring Using GCF:

To factor using a GCF, take the greatest common factor (GCF), for the numerical coefficient. When choosing the GCF for the variables, if all terms have a common variable, take the ones with the lowest exponent.

Example: $9x^4 + 3x^3 + 12x^2$

GCF: Coefficients = 3

Variables (x) = x^2

GCF = $3x^2$

Next, you just divide each monomial by the GCF!

$$\frac{9x^4+3x^3+12x^2}{3x^2} = \frac{9x^4}{3x^2} + \frac{3x^3}{3x^2} + \frac{12x^2}{3x^2} = 3x^2+x+4$$

Put it all together!

Answer = $3x^2(3x^2+x+4)$

Then, check by using the distributive property!

$$(3x^2)(3x^2)+(3x^2)(x)+(3x^2)(4) = 9x^4+3x^3+12x^2$$

Factor each of the following using the GCF and check by using the distributive property:

1) $2a + 2b$

$= 2(a+b)$

Check: $2a+2b$ ✓

2) $18c - 27d$

$= 9(2c-3d)$

check: $18c-27d$ ✓

3) $hb + hc$

$= h(b+c)$

check: $hb+hc$ ✓

4) $p + prt$

$= p(1+rt)$

check: $p+prt$ ✓

5) $10x - 15x^3$

$= 5x(2-3x^2)$

Check: $10x-15x^3$

6) $21r^3s^2 - 14r^2s$ [illegible]

$= 7r^2s(3rs-2)$

check: $21r^3s^2-14r^2s$

7) $c^3 - c^2 + 2c$

$= c(c^2-c+2)$

Check: c^3-c^2+2c ✓

8) $26x^4y - 39x^3y^2 + 52x^2y^3 - 13xy^4$

$= 13xy(2x^3-3x^2y+4xy^2-y^3)$

check: $26x^4y-39x^3y^2+52x^2y^3$

$-13xy^4$ ✓

Factoring Trinomials (Case I):

Case I is when there is a coefficient of 1 in front of your variable^2 term (x^2).

<u>You have two hints that will help you:</u>

1) When the last sign is addition, both signs are the same and match the middle term.
2) When the last sign is subtraction, both signs are different, and the larger number goes with the sign of the middle term.

<u>Examples:</u>

<u>Hint #1:</u>

$x^2 - 5x + 6$

$(x - \ )(x - \ )$

Find factors of 6, w/ sum of 5.

$(x - \underline{3})(x - \underline{2})$

CHECK USING FOIL

$x^2 - 2x - 3x + 6$

$x^2 - 5x + 6$ ✓

<u>Hint #2:</u>

$x^2 + 5x - 36$

$(x - \ )(x + \ )$

Find factors of 36 w/ difference of 5.

$(x - \underline{4})(x + \underline{9})$

CHECK USING FOIL

$x^2 + 9x - 4x - 36$

$x^2 + 5x - 36$ ✓

<u>Factor each trinomial into two binomials and check using FOIL:</u>

1) $a^2 + 3a + 2$ $\quad +2, +1$

$(a+2)(a+1)$

2) $y^2 + 12y + 35$ $\quad +7, +5$

$(y+7)(y+5)$

3) $a^2 + 11a + 18$ $\quad +9, +2$

$(a+9)(a+2)$

4) $a^2 - 8a + 7$ $\quad -7, -1$

$(a-7)(a-1)$

5) $x^2 - 10x + 24$ $\quad -6, -4$

$(x-6)(x-4)$

6) $x^2 - 14x + 49$ $\quad -7, -7$

$(x-7)(x-7)$

7) $x^2 - 6x - 7$ $\quad -7, +1$

$(x-7)(x+1)$

8) $y^2 + 4y - 5$ $\quad +5, -1$

$(y+5)(y-1)$

9) $c^2 + 2c - 35$ $\quad +7, -5$

$(c+7)(c-5)$

10) $z^2 + 9z - 36$ $\quad +12, -3$

$(z+12)(z-3)$

Factoring Trinomials (Case II):

Use Case II when a trinomial has a coefficient other than 1 for the x^2 term.

Let's look at the following example: $6x^2 + 5x - 4$

1) **Look for a GCF:** There is no GCF for this trinomial and the only way this method works is if you take it out right away.

2) **Multiply ac:**

$6x^2 + 5x - 4$

a = 6

c = -4

a*c = -24

3) **Find two numbers that add to b, but multiplies to ac:**

ac

-24

+8 -3

+5

b

+8, -3

4) **Replace the b term (middle term) with the numbers found in step 3.**

$6x^2 \underline{-3x} \ \underline{+8x} - 4$

5) **Factor by grouping:**

$3x(2x-1)+4(2x-1)$

6) **Take out the GCF:**

$(2x-1)(3x+4)$

7) **Foil Check**

$(2x)(3x)+(2x)(4)-1(3x)-1(4)$

$6x^2 + 8x - 3x - 4$

$6x^2 + 5x - 4$ ✓

even if you switch, you get the same answer

$6x^2 \underline{+8x} \ \underline{-3x} - 4$

$2x(3x+4)-1(3x+4)$

$(3x+4)(2x-1)$

observe that you get the same factors

<u>Factor each of the following:</u>

1) $2x^2 + 15x + 7$

ac: 14, b: +15, factors: +1, +14

$= 2x^2 + x + 14x + 7$

$= x(2x+1) + 7(2x+1)$

$= (x+7)(2x+1)$

2) $3x^2 - 5x - 12$

ac: −36, b: −5, factors: −9, +4

$= 3x^2 - 9x + 4x - 12$

$= 3x(x-3) + 4(x-3)$

$= (3x+4)(x-3)$

3) $4x^2 - 16$

$(2x)^2 - 4^2$

$(2x-4)(2x+4)$

4) $7x^2 - 22x + 3$

ac: 21, b: −22, factors: −21, −1

$= 7x^2 - 21x - x + 3$

$= 7x(x-3) - 1(x-3)$

$= (7x-1)(x-3)$

5) $x^3 + 27$

$= x^3 + 3^3$

$= (x+3)(x^2 - (x)(3) + 3^2)$

$= (x+3)(x^2 - 3x + 9)$

6) $8x^3 - 125$

$(2x)^3 - 5^3$

$= (2x-5)((2x)^2 - (2x)(5) + 5^2)$

$= (2x-5)(4x^2 + 10x + 25)$

Factoring Completely:

When asked to factor completely, you will have to use a combination of the methods that we have used previously.

<u>Factor Completely:</u>

1) $4x^2 + 20x + 24$

$= 4(x^2 + 5x + 6)$

$= 4(x+3)(x+2)$

2) $10x^2 - 80x + 150$

$= 10(x^2 - 8x + 15)$

$= 10(x-3)(x-5)$

3) $9x^2 + 90x - 99$

$= 9(x^2 + 10x - 11)$

$= 9(x+11)(x-1)$

Example:	Description of steps:
$2x^5-8x^3=$ $2x^3(x^2-4)=$ $2x^3(x+2)(x-2)$	Step 1: Factor out greatest common factor $(2x^3)$ Step 2: Determine if the remaining binomial is the difference of two squares Step 2: It is the difference of two squares (skip steps 3-4) Step 5: Can it be factored further? No
$3x^4-18x^3+27x^2=$ $3x^2(x^2-6x+9)$ $3x^2(x-3)^2$	Step 1: Factor out greatest common factor $(3x^2)$ Step 2: Determine if the remaining binomial is the difference of two squares: NOT binomial. Step 3: Determine if the remaining trinomial is a perfect square: It seems to be $(x-3)^2$ Step 5: Can it be factored further? No
$6a^2-11a+4=$ $6a^2-3a-8a+4=$ $(6a^2-3a)+(-8a+4)=$ $3a(2a-1)+(-4)(2a-1)=$ $(3a-4)(2a-1)$	Step 1: no GCF Step 2: Not a binomial Step 3: Not a perfect square; factor by AC method (or trial & error). a. Find the product of ac (24). b. Find two numbers whose product is ac (24) and whose sum is b (-11). The two numbers are -8 and -3. c. Rewrite the trinomial so the middle term is the sum of the two numbers found as coefficients. Step 4: Factor by grouping. Step 5: Cannot be factored further.
$xy+8x+3y+24=$ $(xy+8x)+(3y+24)=$ $x(y+8)+3(y+8)=$ $(x+3)(y+8)$	Skip steps 1-3. Step 4: Factor by grouping a. group two terms together b. find GCF of each group c. Use distributive property to "pull out" the common term. d. Rewrite as product of two binomials Step 5: Cannot be factored further
$2ab^5+8ab^4+2ab^3=$ $2ab^3(b^2+4b+1)$	Step 1: Find GCF $(2ab^3)$ Skip step 2 (not a binomial remaining) Step 3-4: Not a perfect square and can't be factored. Step 5: Cannot be factored further.
$x^2+5x+6=$ $(x+3)(x+2)$	Skip steps 1-2 Step 3: Not a perfect square, coefficient of first term is 1, so just reverse FOIL: a. First two terms are x and x b. Last two terms have to multiply to be 6 and sum to be 5. The two numbers are 2 and 3. c. Both signs need to be positive Step 4: Check the OI term to make sure it's correct. It is.

MAT 0024

Ch 13 Factoring Review Worksheet Instructor: C. St.Denis

Factor the following polynomials using the strategy and examples above:

Polynomial:	Factored form:
$12a^2b^2 - 3ab$	GCF: 3ab $3ab(4ab-1)$
$4x^2 - 9 = (2x)^2 - 3^2$	Diff of squares $(2x-3)(2x+3)$
$x^2 - 16y^2 = x^2 - (4y)^2$	Diff of squares $(x-4y)(x+4y)$
$x^2 - 4x + 2xy - 8y$	Grouping $= x(x-4) + 2y(x-4)$ $= (x-4)(x+2y)$
$x^2 - 9x + 20$	$= x^2 - 5x - 4x + 20$ $= x(x-5) - 4(x-5)$ $= (x-4)(x-5)$ 20; −5, −4; −9
$9x^2 - 12x + 4$	$= 9x^2 - 6x - 6x + 4$ $= 3x(3x-2) - 2(3x-2)$ $= (3x-2)(3x-2) = (3x-2)^2$ 36; −6, −6; −12
$8x^3 - x^2$	GCF: x^2 $= x^2(8x-1)$ Note the ^ symbol means to raise as an exponent
$x^2 + 49$	cannot be factored
48; 4, 12; 16 $16x^3 + 16x^2 + 3x$	GCF: x $= x(16x^2 + 16x + 3)$ $= x(16x^2 + 4x + 12x + 3)$ $x(4x(4x+1) + 3(4x+1))$ $x(4x+1)(4x+3)$
$x^2 - 9x + 18$	$= x^2 - 6x - 3x + 18$ $= x(x-6) - 3(x-6)$ $= (x-3)(x-6)$ 18; −6, −3; −9
$6x^2 + 13x + 6$	$= 6x^2 + 9x + 4x + 6$ $= 3x(2x+3) + 2(2x+3)$ $= (2x+3)(3x+2)$ 36; 9, 4; 13

$2x^2+3x-2$	$=2x^2+4x-x-2$ $=2x(x+2)-1(x+2)$ $=(x+2)(2x-1)$ -4; 4, -1; 3
$5x^2-22x-15$	$=5x^2-25x+3x-15$ $=5x(x-5)+3(x-5)$ $=(x-5)(5x+3)$ -75; -25, 3; -22
$3x^3+9x^2-12x$	$=3x(x^2+3x-4)$ $=3x(x+4)(x-1)$
$x^2+3x-28$	$=(x+7)(x-4)$
$x^2-8x+16$ $=x^2+2(x)(-4)+(-4)^2$	Perfect Squares $=(x+(-4))^2=(x-4)^2$
$4x^2-7xy+3y^2$	$=4x^2-4xy-3xy+3y^2$ $=4x(x-y)-3y(x-y)$ $=(x-y)(4x-3y)$
x^3-xy+x^2-y	$=x^3+x^2-xy-y$ $=x^2(x+1)-y(x+1)$ $=(x+1)(x^2-y)$
$8x^2-6x-2$	$=8x^2-8x+2x-2$ $=8x(x-1)+2(x-1)$ $=(x-1)(8x+2)$ -16; -8, $+2$; -6
$x^4-11x^3+24x^2$	$=x^2(x^2-11x+24)$ $=x^2(x-8)(x-3)$
$6x^4y^5-2x^2y^3+14x^3y^4$	GCF: $2x^2y^3$ $=2x^2y^3(3x^2y^2-1+7xy)$