Equations and Inequalities
Presented by QSC

Definitions

- An **expression** is a ____

  ____

The following definitions of an **equation** are equivalent:

- ______________________ set ____________ to each other
- A ______________________ that expresses the ______________

  ______________________________ by connecting them with

  ______________________.

Common Equations/Examples

- Area $A_{rectangle} = bh,\ A_{circle} = \pi r^2$
- Volume $V_{rectangular\ solid} = lwh,\ V_{sphere} = \frac{4}{3}\pi r^3$
- Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Pythagorean Theorem $c^2 = a^2 + b^2$
- Distance $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Circle $(x - h)^2 + (y - k)^2 = r^2$
- Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solving Not Too Scary Equations

- Solve $3x - 4 = 2$
- Solve $4(x - 1) + 2 = 10$
- Solve $6p = 15p - 30$

Absolute Value Equations

- Recall that the absolute value of a number is its distance from 0 on the number line. Absolute values are always greater than or equal to 0.
  Examples: $|-11| =$ _______, $|11| =$ _______.
- For any algebraic expression, x, and any positive real number, n,
  if $|x| = n$, then $x = n$ or $x = -n$.
- How do we solve absolute value equations?
  - Step 1: Isolate the absolute value expression
  - Step 2: Write the equivalent equations. I.e. set the inside expression equal to the positive value it is equal to, and set the inside expression equal to the negative value
  - Step 3: Solve each equation
  - Step 4: Check each solution

- Solve $|3x - 4| = 2$

- Solve $4|x - 1| + 2 = 10$

Linear Inequalities

- A linear inequality is an inequality in one variable that can be written in one of the following forms where a, b, c are real numbers and $a \neq 0$. $ax + b < c,\ ax + b \leq c,\ ax + b > c, ax + b \geq c$
- Solve linear inequalities the way you would a regular linear equation. The difference is we have an inequality symbol instead of = and the solution could be a set of values.
- Properties:
    - Addition/subtraction: whatever you do to one side, do to the other
    - Multiplication/division: whatever you do to one side, do to the other BUT
        - Gotta be careful though! If you multiply or divide by a negative value, we have to flip the symbol of the inequality. Example: 3<4. If we multiply by -1, do we get -3<-4? This is not true, -3>-4. This is why we have to flip the symbol.
- Writing in interval notation:
    - Parenthesis will not include the value
    - Bracket includes the value
    - ∞ or $-\infty$ cannot be included since it is not a finite value that can be reached

In the following exercises, solve, graph, and write solution in interval notation.

- $6p > 15p - 30$

- $9h - 7(h - 1) \leq 4h - 23$

Absolute Value Inequalities (with $<$ or $\leq$)

- For any algebraic expression, x, and any positive real number, n,
  - If $|x| < n$, then $-n < x < n$
  - If $|x| \leq n$, then $-n \leq x \leq n$
- How do we solve absolute value inequalities with $<$ or $\leq$?
  - Step 1: Isolate the absolute value expression
  - Step 2: write the equivalent compound inequality
  - Step 3: Solve the compound inequality
  - Step 4: Graph the solution
  - Step 5: Write the solution using interval notation

In the following exercises, solve, graph, and write solution in interval notation.

- $|2x - 5| \leq 3$

- $|5x + 1| < -2$

Absolute Value Inequalities (with $>$ or $\geq$)

- For any algebraic expression, x, and any positive real number, n,
    - If $|x| > n$, then $x > n$ or $x < -n$
    - If $|x| \geq n$, then $x \geq n$ or $x \leq -n$
- How do we solve absolute value inequalities with $>$ or $\geq$?
    - Step 1: Isolate the absolute value expression
    - Step 2: write the equivalent compound inequality
    - Step 3: Solve the compound inequality
    - Step 4: Graph the solution
    - Step 5: Write the solution using interval notation

In the following exercises, solve, graph, and write solution in interval notation.

- $3|x| + 4 \geq 1$

- $|x - 5| > -2$

Exponential Equations

The following definitions of an exponential equation are equivalent:

- An exponential function is a function whose value is a constant raised to the power of an argument
- An exponential equation is an equation where the variable is located in the exponent position of the equation

Examples:

- $y = 2^x,\ f(x) = 9^{5x-3},\ h(t) = 4^t + 1$

What are they useful for?

- Exponential growth or exponential decay models

- Solve $2^x = 2^7$

- Solve $16^{4x-3} = 32^{\frac{x}{5}}$

- Solve $2^x = 15$