Linear Equations
Presented by QSC

Graphing

A line that goes upward from left to right must have ____________________ slope.

A line that goes downward from left to right must have _______________________ slope.

Recall the equation for slope is:

Two methods for graphing a line are
Method 1:

Method 2:

Let's graph the following lines.

- Graph $y = -\frac{2}{3}x + 5$

- Graph $y = \frac{1}{2}x - 3$

Equations

- Slope-intercept form:
  - Write the equation:
    - $m =$
    - b *is the* _____-value of the _________________ on the line

- Point-slope form
  - Write the equation:

    - $m =$

    - (x_1, y_1) is a __________ on the line

    - We can manipulate point-slope form into slope-intercept form (and vice versa)

- General form
  - Write the equation:

    - $A,\ B,\ C$ *are real numbers*
    - Observe that we can manipulate the general form into other forms of lines as well

- Example 1: Write the equation of the line that has slope of $\frac{3}{5}$ and y-intercept $(0,2)$.

- Example 2: Write the equation of a line that has slope $-\frac{2}{7}$ and goes through the point $(3, -\frac{1}{2})$.

- Example 3: Write the equation of the line that goes through the points $(-1,4)$ and $(2, -1)$.

Parallel or Perpendicular?

- Parallel lines:
  - What does it mean for two lines to be parallel?
    - The lines ______________________________________ each other.
  - How do we know mathematically that two lines will be parallel?
    - The ______________ of the two lines will ____________ each other, i.e. _____________
      - Example: $y =$, $y =$
- Perpendicular lines:
  - What does it mean for two lines to be perpendicular?
    - They ____
  - How do we know mathematically that two lines will be perpendicular?
    - The ______________ of the two lines will ____________________ of each other, i.e.
      - Example: $y =$, $y =$

Determine if the following lines are parallel, perpendicular, or neither:

- $y = \frac{2}{3}x - 4,\ \ y = -\frac{2}{3}x + 3$

Parallel or Perpendicular? Continue determining if the following lines are parallel or perpendicular to each other, or neither.

- $y = -4, \ x = 1$
- $y = \frac{1}{4}x + 5, \ 4y - x = -40$
- $2x - 3y = 12, \ 3x + 2y = 20$

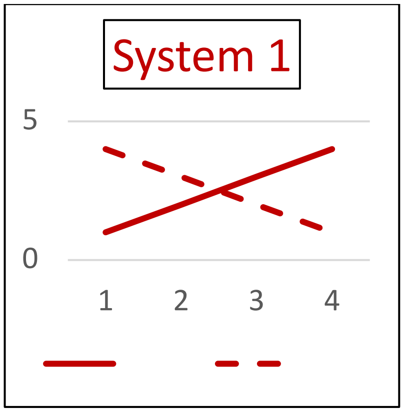
Write one linear equation that is
a) parallel and
b) perpendicular to

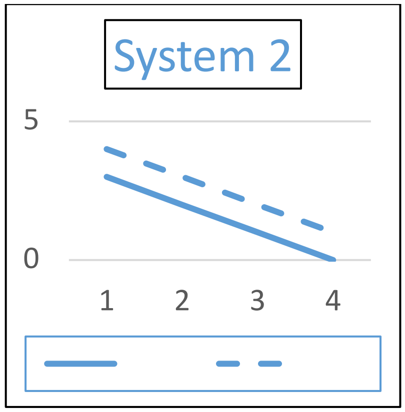
$$y = -2x + 3$$

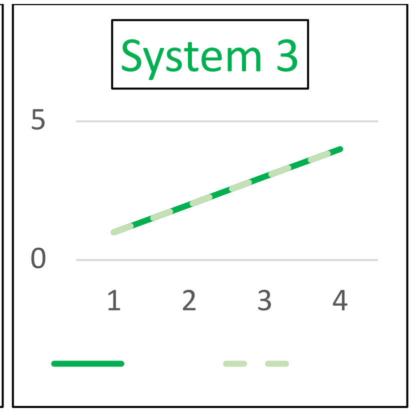
a)

b)

<u>Solving Systems</u>







How many solutions can a system of two linear equations have?

System 1:

- ____________________ because the two lines will ___________________________________.
  How do we tell?
  - When you are solving, you will find ___________________________________.

System 2:

- ____________________ because the two lines will ___________________________________.
  How do we tell?
  - When you are solving, you will end up with __________________________________.

System 3:

- _________________________ because they ____.
  How do we tell?
  - When you are solving, you will end up with __________________________________.

How do we solve for the solutions of a system?
Method 1: Substitution.
Method 2: Elimination/Cancellation.

Find the solution(s) to the following system using substitution

$$\begin{cases} 2x - 4y = -5 \\ 3x + 2y = 8 \end{cases}$$

1. From one of the equations, we solve for one of the variables

2. Then we *substitute* the equation into the other equation to solve for the unknown variable.

3. Then solve for the other variable by plugging into any of the equations. BUT the easiest one is the equation where the variable is already isolated.

Solution:

Find the solution(s) to the following system using elimination/cancellation

$$\begin{cases} 2x - 4y = -5 \\ 3x + 2y = 8 \end{cases}$$

1. Multiply one or both equations by values that will allow one of the variables to be eliminated when you combine/add the two equations together. This will give you one equation with one variable for which you would be able to solve for the variable.

2. Then solve for the other variable by plugging into either of the equations.

Solution:

Find the solution(s) to $\begin{cases} 4x + 3y = 7 \\ 3x + 5y = 8 \end{cases}$

Find the solution(s) to $\begin{cases} 4x + 3y = 7 \\ 3x + 5y = 8 \end{cases}$

Applications

- We can come up with linear equations to help solve for what we are looking for, in math and with real world problems. Here are just a couple of examples.
- Example 1: You have a $50 gift card to your favorite takeout place and you want to order takeout for dinner. Tax is 10% on how much you spend and there is an additional $3 service fee. How much can you spend on food, for the subtotal, so that you do not have to pay out of pocket?
- Example 2: You want to build a fence around the yard to keep the dog in and have 5000 yards of material left for the perimeter. You want the length to be 100 yards longer than twice the width. How long should the length and width of the perimeter of the fencing be?

<u>Absolute Value Equations</u>

- Recall that the absolute value of a number is its distance from 0 on the number line. Absolute values are always greater than or equal to 0. Examples: $|-11| = 11$, $|11| = 11$.
- For any algebraic expression, x, and any positive real number, n, if $|x| = n$, then $x = n$ or $x = -n$.
- How do we solve absolute value equations?
  - Step 1: Isolate the absolute value expression
  - Step 2: Write the equivalent equations. I.e. set the inside expression equal to the positive value it is equal to, and set the inside expression equal to the negative value
  - Step 3: Solve each equation
  - Step 4: Check each solution
- Solve $|3x - 4| = 2$

- Solve $4|x - 1| + 2 = 10$

Linear Inequalities

- A linear inequality is an inequality in one variable that can be written in one of the following forms where a, b, c are real numbers and $a \neq 0$. $ax + b < c,\ ax + b \leq c,\ ax + b > c, ax + b \geq c$
- Solve linear inequalities the way you would a regular linear equation. The difference is we have an inequality symbol instead of = and the solution could be a set of values.
- Properties:
  - Addition/subtraction: whatever you do to one side, do to the other
  - Multiplication/division: whatever you do to one side, do to the other BUT
    - Gotta be careful though! If you multiply or divide by a negative value, we have to flip the symbol of the inequality. Example: 3<4. If we multiply by -1, do we get -3<-4? This is not true, -3>-4. This is why we have to flip the symbol.
- Writing in interval notation:
  - Parenthesis will not include the value
  - Bracket includes the value
  - ∞ or $-\infty$ cannot be included since it is not a finite value that can be reached

In the following exercises, solve, graph, and write solution in interval notation.

- $6p > 15p - 30$

- $9h - 7(h - 1) \leq 4h - 23$

<u>Absolute Value Inequalities</u>

- For any algebraic expression, x, and any positive real number, n,
  - If $|x| < n$, then $-n < x < n$
  - If $|x| \leq n$, then $-n \leq x \leq n$
- How do we solve absolute value inequalities with $<$ or $\leq$?
  - Step 1: Isolate the absolute value expression
  - Step 2: write the equivalent compound inequality
  - Step 3: Solve the compound inequality
  - Step 4: Graph the solution
  - Step 5: Write the solution using interval notation

In the following exercises, solve, graph, and write solution in interval notation.

- $|2x - 5| \leq 3$

- $|5x + 1| < -2$

- For any algebraic expression, x, and any positive real number, n,
  - If $|x| > n$, then $x > n$ or $x < -n$
  - If $|x| \geq n$, then $x \geq n$ or $x \leq -n$
- How do we solve absolute value inequalities with $>$ or $\geq$?
  - Step 1: Isolate the absolute value expression
  - Step 2: write the equivalent compound inequality
  - Step 3: Solve the compound inequality
  - Step 4: Graph the solution
  - Step 5: Write the solution using interval notation

In the following exercises, solve, graph, and write solution in interval notation.

- $3|x| + 4 \geq 1$

- $|x - 5| > -2$

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