Functions
Presented by QSC

Definition

- Function: a relation between two sets of variables where each ____________________ variable is assigned to ________________________________ variable. So if your independent variable is x and your dependent variable is y,

  ____ assigned to it.

- Every _____________ (______________________ variable) will have

  ____.

- A function and variables can be denoted in many ways.
  - Example: We can call the function y and the independent variable x and our function is $y = x^2 + 1$
  - Example: We can call the function f, and the independent variable is x. Our function is $f(x)$ and $f(x) = x - 8$.
  - Example: If our function is $h(t)$, t is the independent variable and we call our function h.
    - We can represent the previous functions with $h(t)$ instead. I.e. $h(t) = t^2 + 1$ or $h(t) = t - 8$ instead.

Method of 4: Function Representation

- There are four ways for the representation of a function:
  - Numerically – a function is represented using a table of values or chart
  - Verbally – word description is used to represent the function
  - Symbolically/algebraically – a function is represented using a mathematical model
  - Visually – the function is shown using a graph
    - Note that we can use the four methods above to represent a function
- How do we tell if something will be a function?
  - Numerically – when we look at a table or chart, what would we need to be a function?
    - Every independent variable will only have ________________________ dependent variable.
  - Verbally – when we are told about a function, what would we need to be a function?
    - Similar to numerically, we will not have any independent variables

      assigned to ______________________________ dependent variable.

- Symbolically/algebraically – when we look at an equation, what would we need to be a function?
  - We will ___________________________________ dependent value assigned for each independent value. We will not have more than one output.
- Visually – when we look at the graph, how do we tell if it is of a function?
  - Every independent variable that is graphed will ______________________ corresponding dependent variable.
    - We can do a visual test on the graph. More specifically, the

      ____. If you have

      ___________________________________ line that crosses the

      curve ______________________________________, it fails to be a function.

<u>Function or Not?</u>

Numerically – when we look at a table or chart, what would we need to be a function?

Every independent variable will ______________________________ dependent variable.

Which of the following is a function?

x	$f(x)$
-1	8
0	5
1	-3
2	0.5
3	5

Input	Output
-1	8
0	5
1	-3
2	0.5
3	2

Input	Output
-1	8
0	5
1	-3
2	0.5
-1	2

Symbolically/algebraically – when we look at an equation, what would we need to be a function?
We will ____ dependent value assigned for each

independent value. We will __________________________ than one output.

Which of the following are functions?

- $y = x + 5$
- $f(x) = x^2 + x - 3$
- $x = y^2 + 1$
- $x^2 + y^2 = 1$

Visually – when we look at the graph, how do we tell if it is of a function?

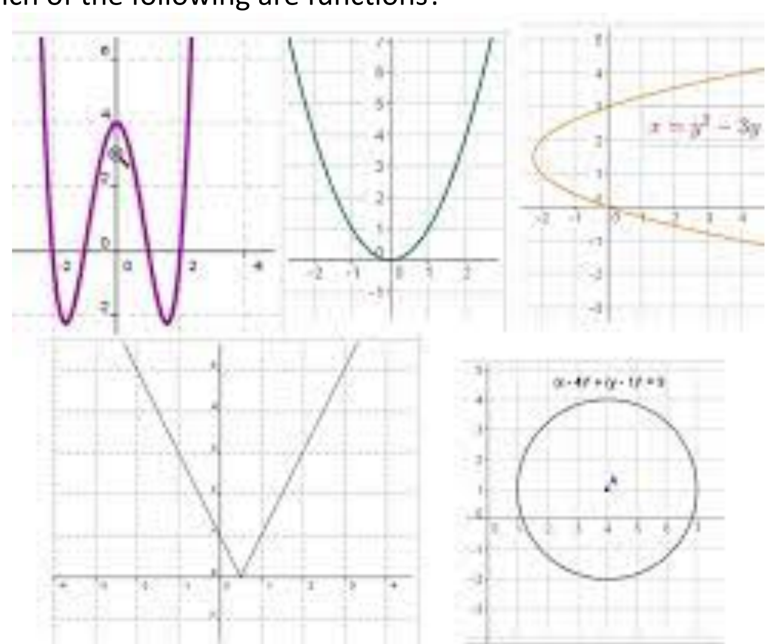
Every independent variable that is graphed will ___________________________________ corresponding dependent variable.

We can do a visual test on the graph. More specifically, the ________________line

test. If you have _________________________________ line that crosses the curve

_______________________________________, it fails to be a function.

Which of the following are functions?



Evaluation

- Every input will have an output, which we can determine using at least one of the four methods (numerical, verbal, symbolic/algebraic, visual).
- Example: What is $f(3)$ if $f(x) = x^2 - x - 6$?
    - Symbolic/algebraically?
    - Visually?
    - Numerically?
    - Verbally?
- Let c be an unknown constant, and $f(x)$ be the function above. What is $f(c)$?
- Let $f(x)$ be the function above. What is $f(x+h)$? What is $f(x) + h$? Are they the same?

Domain and Range

- Domain: the set of all _______________ values of the function for which the function is defined. The set of input values for which the function can take in and it makes sense.

- Range: the set of ______________ values of the function.
- Examples:
  - What is the domain and range of $f(x) = x^2 - x - 6$?

  - What is the domain and range of $g(t) = \sqrt{t-3}$?

- How can we determine domain and range without their graphs?
  - We can think about which input values the function can take in without any issues. Or think about the restrictions for which the inputs cannot take on and exclude those values from the real numbers.
  - Then for those inputs, consider what outputs the function would spit out.

Algebraic Operations

- We can perform the usual algebraic operations on functions, such as addition, subtraction, multiplication and division.
  - We do this by performing the operations with the function outputs, defining the result as the output of our new function.
  - If we have two functions $f(x),\ g(x)$, then
    - $(f+g)(x) = f(x) + g(x)$
    - $(f-g)(x) = f(x) - g(x)$
    - $(fg)(x) = f(x)g(x)$
    - $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
      - Note that we can perform a combination of algebraic operations on multiple functions. For example if we have three functions $f(x),\ g(x), h(x)$, we can obtain a new function $j(x) = \frac{f(x)}{g(x)-h(x)}$.

- Given $f(x) = 2x^2 + 1$ and $g(x) = 3x - 5$, find
  - $f + g,\ f - g,\ fg, \frac{f}{g}$

  - $g + f, g - f, gf, \frac{g}{f}$. Are these the same as the previous functions?

<u>Composition</u>

Composition of functions is an operation that takes two functions to generate a new function. If you have two functions $f,\ g$, you can compose the two to get a new function, i.e. $(f \circ g)(x) = f(g(x))$. You can also think of it as the input of one function being another function.

- Find $(f \circ g)(x),\ (g \circ f)(x), (f \circ f)(x),\ (g \circ g)(x), (f \circ g)(-2)$ if

$$f(x) = x^2 + 5x,\ g(x) = 4x - 1$$

- Suppose $h(x) = \sqrt{x}$ and $f(x)$ and $g(x)$ are the same functions above. Find $(f \circ g \circ h)(x)$

Inverse

An inverse function is a function that undoes/reverses the action of the function. If you compose a function with its inverse function, or vice versa, you will get ____________________ ____. I.e. if your function is f and your inverse function is g, then $(f \circ g)(x) = (g \circ f)(x) = x$. We can also denote a function's inverse as $f^{-1}(x)$ if the function is f. Note that $f^{-1}(x) \neq \frac{1}{f(x)}$

How do we find a function's inverse? ____.

What would a function's inverse look like visually/graphically? Since we are switching x and y to find the inverse of a function, we switch the x and y coordinates of every point of the function.

This means visually, the inverse looks like ____ ____________________________________.

In order to have an inverse function, the function must be ______________________. This means for every _____________________, there is at most one ___________________.

How can we test that a function is one-to-one? We can do a visual test to determine if a function will be one-to-one. More specifically, the _________________________. If you draw __________________________________, the function that goes through ___________________ ___________________________ is a one-to-one function.

x	$f(x)$
-1	8
0	5
1	-3
2	0.5
3	2

- Find $f^{-1}(2)$, $f^{-1}(3)$, $f^{-1}(5)$ using the table above
- Confirm the functions

$$f(x) = \sqrt[3]{x-1},\ g(x) = x^3 + 1$$

are inverses of each other.

- Find $f^{-1}(x)$ for each function:
  - $f(x) = 3 - x$

- $f(x) = \frac{x}{x+2}$

- $f(x) = x^3 + 1$

<u>Applications</u>

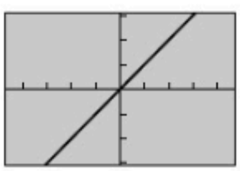
- We can come up with functions for real-life examples.
- Example 1 (exponential growth): You put $1000 into a bank account that has an interest rate of 2.5% and compounds interest on a monthly basis. This can be modeled by the function $A(t) = 1000\left(1 + \frac{0.025}{12}\right)^{12t}$, where t represents time in years. Figure out how much money you would have in your account if you do not make any withdrawals from the account in half a year, 1 year, and 2 years.

- Example 2 (distance): A car travels at a constant speed of 60 miles per hour. The distance the car travels in miles is a function of time, t, in hours given by $d(t) = 60t$. How many miles does the car drive in 3 hours?

- Example 3 (temperature): The United States is one of the few places that uses Fahrenheit for temperature. Suppose we go on vacation in Europe and want to convert from C degrees to F Fahrenheit. We use the formula $F(C) = \frac{9}{5}C + 32$.
    - How many degrees in Farenheit is 15°C? What about 25°C?
    - Find the inverse function, and explain its meaning.

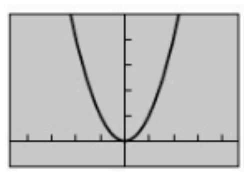
Transformations

Here are some graphs of the "basic" functions we should know.



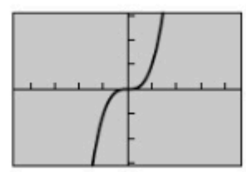
[–4.7, 4.7] by [–3.1, 3.1]

Identity Function
$f(x) = x$
Domain = $(-\infty, \infty)$
Range = $(-\infty, \infty)$



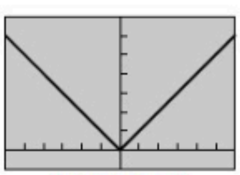
[–4.7, 4.7] by [–1, 5]

Squaring Function
$f(x) = x^2$
Domain = $(-\infty, \infty)$
Range = $[0, \infty)$



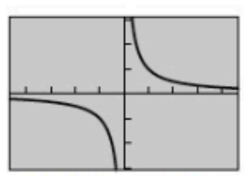
[–4.7, 4.7] by [–3.1, 3.1]

Cubing Function
$f(x) = x^3$
Domain = $(-\infty, \infty)$
Range = $(-\infty, \infty)$



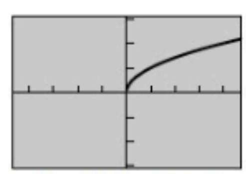
[–6, 6] by [–1, 7]

Absolute Value Function
$f(x) = |x| = \text{abs}(x)$
Domain = $(-\infty, \infty)$
Range = $[0, \infty)$



[–4.7, 4.7] by [–3.1, 3.1]

Reciprocal Function
$f(x) = \frac{1}{x}$
Domain = $(-\infty, 0) \cup (0, \infty)$
Range = $(-\infty, 0) \cup (0, \infty)$



[–4.7, 4.7] by [–3.1, 3.1]

Square Root Function
$f(x) = \sqrt{x}$
Domain = $[0, \infty)$
Range = $[0, \infty)$

Knowing the basic functions allows us to transform these functions easily.

Horizontal transformations effect the independent variable.
$f(x+c)$ is a horizontal shift to the left by c | $f(x-c)$ is a shift to the right by c
$f(bx)$ is a horizontal stretch if b is between 0 and 1 | if $b > 1$, then the function shrinks
$f(-x)$ is a reflection across the y-axis (negating the x-values)
Vertical transformations effect the dependent variable, i.e. the entire output.
$f(x) + d$ is a vertical shift up by d | $f(x) - d$ is a shift down by d
$af(x)$ is a vertical stretch/shrink
$-f(x)$ is a reflection across the x-axis (negating the y-values)
How do we determine the order of transformations? We can use order of operations (PEMDAS) based on the function to help determine the order of transformations.
If a function has all of these transformations, it can be combined into a general form where the function is of the form $af\big(b(x+c)\big) + d$.

- Given a function $f(x)$, describe how the function is transformed when:
    - $y = f(x+4) - 1$
    - $g(x) = -f(3x)$
- Describe how the formula is a transformation of a basic function:
    - $a(x) = \sqrt{-x+4}$
    - $h(x) = -2|x-4| + 3$

- $g(x) = 5(x+3)^2 - 2$

Workshop Survey:

QSC Workshops: