

Trigonometry for Calculus

Presented by the Quantitative Success Center

RECIPROCAL IDENTITIES

$$\sin(\theta) = \frac{1}{\csc(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

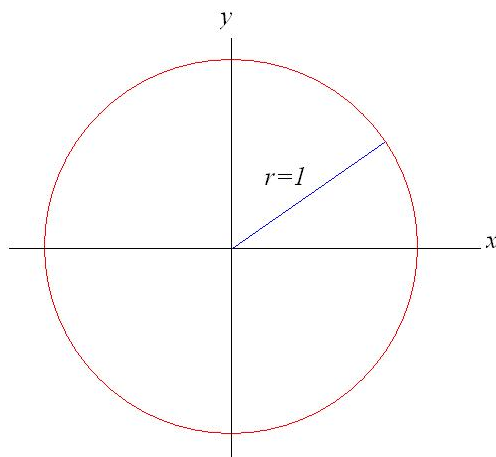
$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Remember: ALL identities can be written in terms of _____ & _____.

A. Unit circle

Recall: $x = \cos \theta$, $y = \sin \theta$

where θ is the angle you take going counterclockwise from the positive x -axis.

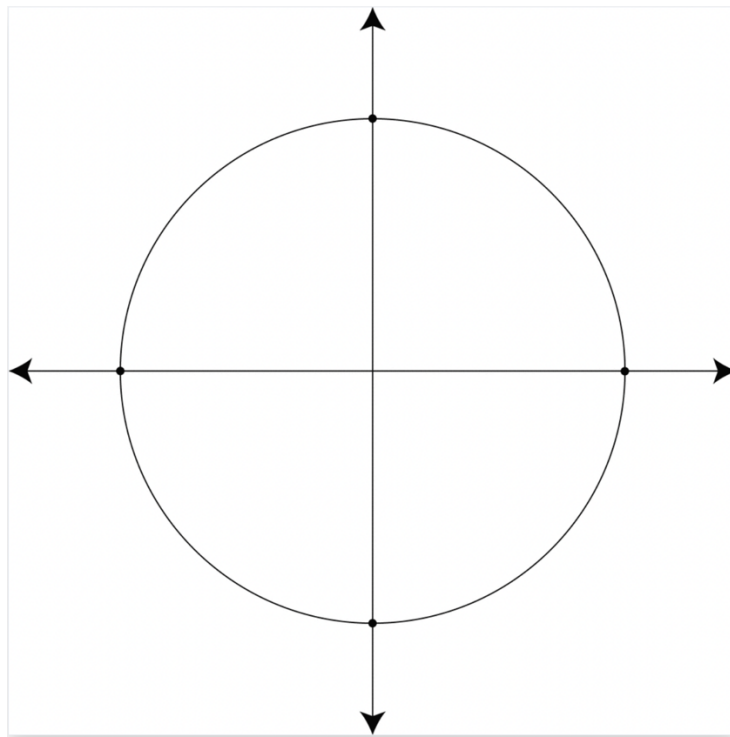


| Quadrant | Are the x -values positive or negative? | Are the y -values positive or negative? | Are the $\frac{y}{x}$ values positive or negative? |
|------------|---|---|--|
| I | | | |
| II | | | |
| III | | | |
| IV | | | |

Find the sign of each trigonometric function in the respective quadrant.

| Quadrant | $\cos x$ | $\sin x$ | $\tan x$ | $\sec x$ | $\csc x$ | $\cot x$ |
|----------|----------|----------|----------|----------|----------|----------|
| I | | | | | | |
| II | | | | | | |
| III | | | | | | |
| IV | | | | | | |

We can conclude - All Students Take Calculus:



B. Pythagorean Identities

(Manipulating $\cos^2 \theta + \sin^2 \theta = 1$ to get the other identities)

- a. Since $x^2 + y^2 = 1$ on the unit circle, we get $\cos^2 \theta + \sin^2 \theta = 1$
- b. Let's divide our identity from part a) by $\cos^2 \theta$ and see what we get:

- c. Let's divide our identity from part a) by $\sin^2 \theta$ and see what we get:

- d. We can conclude with the three trigonometric identities from (a)-(c):

| |
|--|
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| |
| |

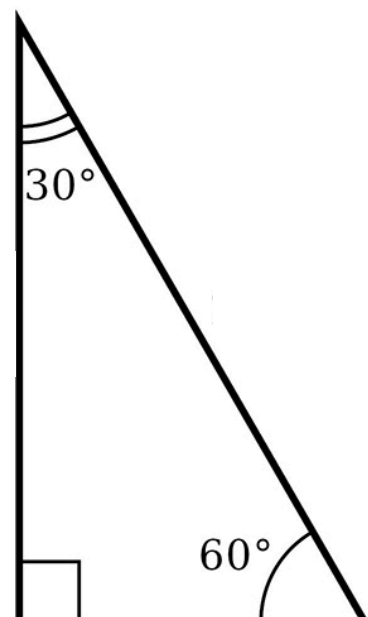
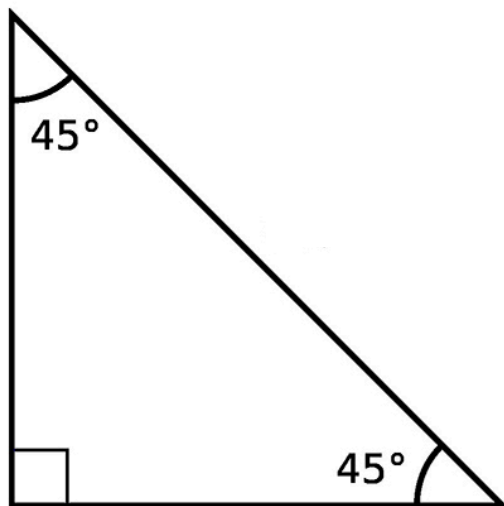
C. Converting between degrees and radians.

Recall $\pi = 180^\circ$

| Degrees $^\circ$ | Radians |
|------------------|-----------------|
| 30° | |
| | $\frac{\pi}{4}$ |
| 60° | |
| | $\frac{\pi}{2}$ |

| Degrees $^\circ$ | Radians |
|------------------|-------------------|
| 120° | |
| 135° | |
| | $\frac{4\pi}{3}$ |
| | $\frac{11\pi}{6}$ |

D. Special Right Triangles & Reference Angles



Recall: SOH CAH TOA. $\sin x = \frac{\textit{opposite}}{\textit{hypotenuse}}$, $\cos x = \frac{\textit{adjacent}}{\textit{hypotenuse}}$, $\tan x = \frac{\textit{opposite}}{\textit{adjacent}}$

a. What is the reference angle for $\frac{2\pi}{3}$?

b. What is the reference angle for $\frac{3\pi}{4}$?

c. What is the reference angle for 240° ?

d. What is the reference angle for 330° ?

More Practice:

Find the exact values using unit circle/triangles/identities.

1. 30°

| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

2. $\frac{\pi}{4}$

| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

3. 60°

| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

4. $\frac{\pi}{2}$

| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

5. 120°

| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

6. $\frac{4\pi}{3}$

| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

7. 135°

| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

8. $\frac{11\pi}{6}$

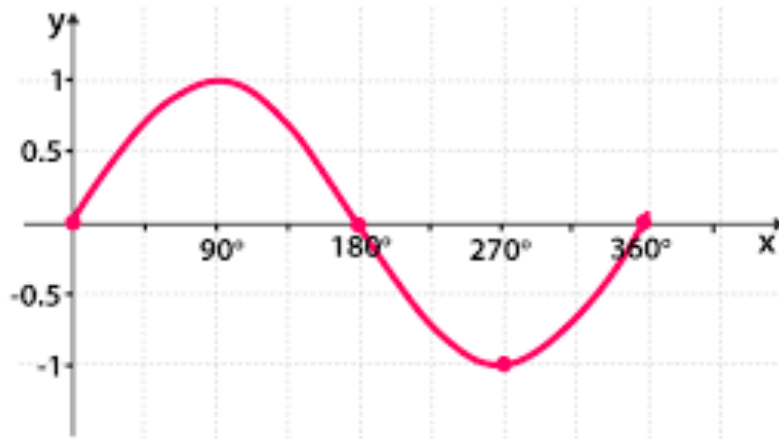
| | |
|------------|------------|
| $\cos x =$ | $\sec x =$ |
| $\sin x =$ | $\csc x =$ |
| $\tan x =$ | $\cot x =$ |

E. Determine the exact values for the following trigonometric functions when the exact angles are not given:

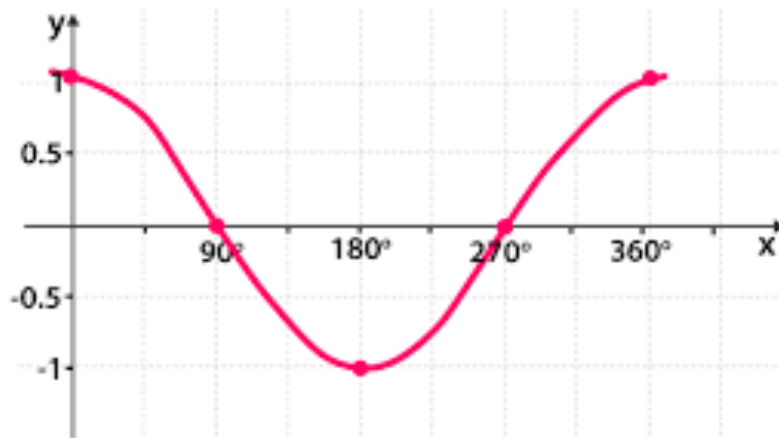
| $\sin x$ | $\cos x$ | $\tan x$ |
|---------------|---------------|---------------|
| $\frac{2}{3}$ | | |
| | | $\frac{1}{4}$ |
| | $\frac{4}{5}$ | |

F. Graphs of sine and cosine to also help determine exact values

a. Graph of $y = \sin x$



b. Graph of $y = \cos x$



G. More resources

- For the review and/or its solutions, visit qsc.whittier.domains and click on “Workshops”
- <http://tutorial.math.lamar.edu/classes/calci/calci.aspx>

Workshop Survey



<https://forms.gle/y6u2s8TQymPYA1vN9>